

Mad Veterinarian Puzzles

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These problems have been adapted from <http://www.bumblebeagle.org/madvet/index.html>

<http://www.mathteacherscircle.org/assets/session-materials/GAbramsMadVetpuzzles.pdf>

<http://wordplay.blogs.nytimes.com/2011/06/27/numberplay-the-mad-vet>

Puzzle 1

A Mad Veterinarian has created three animal transmogrifying machines.

If you put two cats into Machine #1 and press the TRANSMOGRIFY button, the machine converts the two cats into a dog. Machine #2 can convert two dogs into a dog, and Machine #3 can convert a cat and a dog into a cat. Each machine can also operate in reverse, for example, the third machine can convert a cat into a dog and a cat.

Question 1a. If the Mad Vet starts with three cats and one dog, can she end up with only one cat? What if the vet started with four cats and two dogs?

Question 1b. For which starting collections of animals is it possible to end up with only one cat? Why?

Question 1c. For which starting collections of animals is it possible to end up with only a pack of dogs? Why?

Puzzle 2

Another Mad Vet has these three machines:

Machine #1 turns one cat into one dog.

Machine #2 turns one dog into one cat, one dog, and one mouse

Machine #3 turns one mouse into one cat and one dog

This Mad Vet has one cat.

Question 2a. Can she convert it into two cats? (If so, show how. If not, explain why not.)

Question 2b. Can she convert it into three cats? (If so, show how. If not, explain why not.)

Question 2c. Can she convert it into three cats and one dog? (If so, show how. If not, explain why not.)

Question 2d. Can she convert it into one mouse? (If so, show how. If not, explain why not.)

Question 2e. Using these three Mad Vet machines, how many different "classes" of animals are there? Describe all of the classes.

Question 2f.

A friend of this Mad Vet brings her, as a birthday present, a cat and a mouse. Our Mad Vet prefers to have just one cat. Can she transmogrify her new collection of animals back to the way things were when she only had one cat?

Question 2g. Does your answer to the previous question depend on the fact that our Mad Vet started with one cat? That is, if she started with ANY collection of animals, and her friend brought her a cat and a mouse, would she be able to transmogrify her collection back to the original collection?

Question 2h. (See Question 2d) The Mad Vet is looking to close down the business, but she'd like to finish this Mad Vet business owning only one mouse (and no other animals). From your answer to Question 2.4, you know she can't simply use the cat she has to produce one mouse. So she'll go to the pet shop down the street, buy some animals, and then use those animals together with her one cat to produce a mouse. What animals should she buy at the pet shop?

Question 2i. By adding animals from the pet shop to the cat she currently owns, is it possible for the Mad Vet to finish her Mad Vet business owning whatever collection of animals she'd like?

Question 2j. Does your answer to the previous question depend on the fact that she happens to currently own one cat?

Puzzle 3

Yet another Mad Vet has these three machines:

Machine #1 turns one cat into one dog and one mouse

Machine #2 turns one dog into one cat and one dog

Machine #3 turns one mouse into one cat and one mouse

This Mad Vet has one cat.

Question 3a. Can he convert it into two cats? Three cats? Any number of cats? (If so, show how. If not, explain why not.)

Question 3b. Can he convert it into one dog? (If so, show how. If not, explain why not.)

Question 3c. Can he convert it into one mouse? (If so, show how. If not, explain why not.)

Question 3d. Using these three Mad Vet machines, how many different "classes" of animals are there? Describe all of the classes.

Question 3e. A friend of this Mad Vet insists on giving the Mad Vet a gift of animals for his birthday. Our Mad Vet really prefers to have one cat. So as not to offend his friend, what collection of animals should the Mad Vet tell his friend to bring his for a birthday present?

Question 3f. Does your answer to the previous question depend on the fact that our Mad Vet started with one cat? That is, if he started with ANY collection of animals, and his friend brought her a cat and a mouse, would he be able to transmogrify her collection back to the original collection?

Question 3g. The Mad Vet is looking to close down the business, but he'd like to finish his Mad Vet business owning only one mouse (and no other animals). From a previous question, you know he can't simply use the cat she has to produce one mouse. So, he'll go to the pet shop down the street, buy some animals, and then use those animals together with his one cat to produce a mouse. What animals should he buy at the pet shop?

Question 3h. By adding animals from the pet shop to the cat he currently owns, is it possible for the Mad Vet to finish his Mad Vet business owning whatever collection of animals he'd like?

Question 3i. Does your answer to the previous question depend on the fact that he happens to currently own one cat?