The Teacher's Circle

# Digit Fidgets

Sam Vandervelde
samv@mandelbrot.org
June 18, 2007

Our motivational question for the day was "In the decimal representation for 1/2008, how many digits beyond the decimal point must we write before the digits begin to repeat?" Rather than launch a direct assault on the problem using calculators, we decided to try some simpler cases to understand how the number in the denominator influences the answer. The following sequence of problems outlines one approach to determining the answer.

**Problems.**

1. Working together in groups of three, make a table showing the decimal representations for the fractions 1/2, 1/3, . . . , through 1/25. In each case include the digits up to the point where the digits begin to repeat, or where the decimal terminates. If any of these fractions are too much trouble just skip them for now.

2. Depending upon the denominator used, we obtain different sorts of behavior in the various decimal expansions. Describe the sorts of results that are possible.

3. More precisely, make a conjecture as to which denominators yield a terminating decimal. How does the value of the denominator determine the number of digits before the decimal terminates? Based on your answer, predict the number of digits in the decimal for 1/128.

4. The remaining decimals all contain repeating strings of digits. But some of them begin with a few extra "random" digits at the start before entering the repeating phase, while others consist of nothing but the same repeating string of digits right from the start. How can we predict which denominators give which sort of behavior?

5. Let's take a closer look at the instances in which there are a few extra "random" digits at the start. How can we predict the number of non-repeating initial digits based on the denominator? In particular, how many such digits will there be for 1/2008?

6. We now turn our attention to the cases in which the decimal is a purely repeating string of digits. How is the number of digits in the repeating portion related to the denominator?

7. Once you have settled on a response to the previous problem, try coming up with an answer to the same question in the case of decimals which involve a few initial non-repeating digits. In particular, what fraction should we examine in order to understand the behavior of the decimal expansion of 1/2008?

8. The time has come to deal with the problematic fractions in our table, such as 1/17, 1/19, and 1/23. How is it possible to find the entire repeating portion of the decimal with a calculator which displays only eight or ten digits at best? (Hint: what do you notice about the decimal representations of the fractions 1/7, 2/7, . . . , 6/7? How will this help?)

9. Finally, put everything together to answer the original question which motivated this whole exploration in the first place.