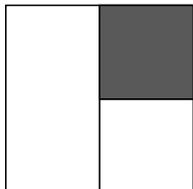


# Fraction Conundrums

## Warm Up Questions

For each of the following questions, consider the following questions:

- a. What is your answer to the question and how would you explain it to someone who knows nothing about fractions?
- b. What incorrect answers do you think people might give and why are those answers reasonable?
  1. What fraction is shown below?



2. What does  $\frac{5}{8}$  mean?
3. What does  $\frac{5}{3}$  mean?
4. Which is larger,  $\frac{1}{8}$  or  $\frac{1}{3}$ ?
5. Which is larger,  $\frac{2}{3}$  or  $\frac{3}{4}$ ?
6. Draw a picture representing  $\frac{3}{4}$ .
7. Draw a picture representing  $\frac{5}{3}$ .
8. Sketch part of a ruler and show precisely where  $\frac{5}{8}$  should be located.
9. Sketch part of a ruler and show where  $\frac{17}{4}$  should be located.
10. How could you divide 3 cookies evenly among 4 people?

## Create Your Own Scenario

For each of the following computations, give a scenario or story problem that could be solved using that computation. Draw a picture that shows the situation and the answer clearly.

a)  $9\frac{7}{10} + 4\frac{3}{20}$

c)  $2\frac{1}{3} \times 1\frac{1}{2}$

b)  $4\frac{1}{4} - 1\frac{1}{2}$

d)  $2\frac{1}{2} \div \frac{5}{8}$

## Level 1

1. Maurice and his grandmother made a rectangular quilt using 24 square fabric pieces. Half of the quilt was red, one sixth of the quilt was white, and the rest was black. Draw some designs to show how the quilt might have looked. Which one is your favorite? Use pictures, numbers, and words to explain your designs.
2. Cindy baked a batch of 12 cookies. She placed  $\frac{1}{6}$  of the cookies on a red plate and  $\frac{3}{6}$  on a blue plate. She left the rest of the cookies on the cookie tray. Which plate had the most cookies and how can you tell? How many cookies were left on the cookie tray?
3. Cedric baked a batch of 12 cookies. He placed  $\frac{1}{4}$  of the cookies on a red plate and  $\frac{1}{3}$  of the cookies on a blue plate. He left the rest of the cookies on the cookie tray. Which plate had the most cookies and how can you tell? How many cookies were left on the cookie tray?
4. Jennifer had \$30 to spend on herself. She spent  $\frac{1}{5}$  of the money on a sandwich,  $\frac{1}{6}$  of the original amount of money for a ticket to a museum, and  $\frac{1}{2}$  of the original amount of money on a book. How much money does Jennifer have left over?
5. Stephanie's garden is  $\frac{2}{3}$  of an acre. She decides to plant a different vegetable on each  $\frac{1}{6}$  acre. How many different vegetables can she plant?
6. Gina drove for  $\frac{1}{3}$  of the trip, and Mark drove for  $\frac{1}{6}$  of the trip. Joe and Katie divided the rest of the driving evenly between them. If the entire trip was 132 miles, how many miles did Katie drive?
7. Each box is  $5\frac{3}{8}$  inches wide. If a shelf is 48 inches long, how many boxes will fit side by side on the shelf without dangling over the edges?
8. One lap around the block is  $\frac{7}{10}$  of a mile. If I run 4 laps, how far did I run?
9. If 1 lap around your block is  $\frac{5}{8}$  of a mile, how many laps do you need to run to reach  $7\frac{1}{2}$  miles?
10. A room is 12 feet long and Wanda wants to place tiles on the wall as a decoration. If each tile is  $\frac{2}{3}$  ft wide, how many tiles will she need to complete one row?
11. Sarah mixed  $2\frac{1}{2}$  cups of chopped apples,  $1\frac{1}{8}$  cups of grapes and  $1\frac{1}{4}$  cups of chopped pears to make a fruit salad. After setting aside  $1\frac{1}{2}$  cups of fruit salad for lunch today, she divided the rest of the salad equally into 3 containers. How much fruit salad is in each container?

## Level 2

1. A theoretical frog is catching a hypothetical fly. Measure and cut a rectangular tongue for the frog which is exactly  $\frac{5}{3}$  of a foot long and 3.5 centimeters wide. Measure a second tongue which is  $\frac{1}{4}$  of a yard long and 42 millimeters wide. Measure a third tongue which is 0.4 meters long and  $2\frac{3}{8}$  inches wide. You may need to use glue or tape.
2. Three tenths of the wooden toys at Art's toy workshop were painted blue and one fourth of them were painted green. Half of the remaining toys were painted red and half were painted yellow. If 300 toys are blue, how many are there of each of the other colors?
3. The students at Meadow's Edge Elementary School have been learning to make nutritious lunch choices. The students learned that canned corn is a starch and does not really count as a serving of vegetables. Today, students could choose either canned corn or a fresh spinach salad as their vegetable.  $\frac{7}{10}$  of the students chose the healthier option, but the other 90 students opted for the corn anyway because they do not like the taste of spinach. How many students are there at Meadow's Edge Elementary School?
4. Sybil the Snail is crawling in a race, and has just reached the half way point. If Sybil crawls 12 more inches, she will have completed  $\frac{7}{10}$  of the race. How many inches long is the full race course?
5. It costs \$1.92 for a whole box of Cheerios containing 16 cups of cereal. You use  $\frac{1}{2}$  of a cup of Cheerios in your trail mix. What is the cost of the Cheerios that you used in your mix?
6. I love to listen to music, and I just got a new MP3 player so that I can take music with me everywhere I go. My new MP3 player can hold about  $14\frac{1}{2}$  hours of music. If the average length of the songs on my play list is 3 minutes and 20 seconds, how many songs can my MP3 player hold?
7. On Monday, you start work at 10:35 AM and finish work at 3:20 PM. Your rate of pay is \$9.50 per hour. How much money should you be paid for Monday's work?
8. Julio is training for a marathon and wants to know what fraction of his time he spends running each day. This morning, he went for a quick run that lasted two-thirds of an hour. He went for a longer run from 11:40 AM to 2:05 PM. In the evening, he ran for 1.25 hours. What fraction of this day did he spend running?
9. My recipe calls for 6 teaspoons of sugar to make 8 apple crisp dessert cups. How many teaspoons of sugar do I need to make 16 apple crisp dessert cups? How much sugar do I need for 12 dessert cups? For 10 dessert cups?
10. The store has milk on sale for \$1.80 a gallon, which is 75% of the usual price. What is the usual price?
11. My father is a teacher and he has a stack of papers to grade this weekend. It took him  $2\frac{1}{2}$  hours to grade  $\frac{2}{5}$  of the papers this afternoon. He started grading at 12:30 pm. Will he finish by dinner time at 6 pm? If so, what time will he likely finish? If not, how many more hours of work will he have?

### Level 3

1. Lisa and Colin go out to eat at a restaurant and ask the waiter to give them separate bills. Lisa's bill totals \$10. Lisa leaves one tenth (or 10%) of that amount as a tip. Colin's bill totals \$15. Colin leaves a tip that is one fifth (or 20%) of his bill.

Lisa adds the two fractions and thinks that they are giving the waiter a tip of  $\frac{3}{10}$  of their combined bill. Colin disagrees. He says that the waiter receives a 15% tip for their two bills combined because that is the average of 10% and 20%.

- a. Who is correct? If either of them made a mistake, why didn't the approach work?
  - b. What fraction of their combined bill did the two friends leave as a tip? (Explain your solution using pictures, numbers, and complete sentences.)
2. Half Max's allowance is equal to  $\frac{2}{5}$  of Donna's allowance. Keri's allowance is 3 times as much as Max's. In all, their allowances add up to \$21. How much is Max's allowance?
  3. Lydia made some cookies, of which  $\frac{1}{3}$  were chocolate chip,  $\frac{3}{4}$  of the remainder were gingersnaps, and the rest were peanut butter. If Lydia made 4 dozen cookies altogether, how many peanut butter cookies did she make?
  4. The flu has hit Boland Elementary. In Ms. Hefling's class,  $\frac{1}{3}$  of the students were absent today. In Ms. Dempsey's class,  $\frac{2}{5}$  of the students were absent. There were 8 students absent in each class. If everyone was present, how many students would there be in each class?
  5. At Middletown Elementary School,  $\frac{4}{9}$  of the lower elementary students are girls, and  $\frac{2}{3}$  of the girls are lower elementary students.
    - a. Are there more girls or more lower elementary students at the school?
    - b. There are 90 students at Middletown Elementary School, and  $\frac{1}{2}$  of them are lower elementary students. How many of the students are boys?
  6. Cathy and Karl Brown own a dairy farm and they are planning for the winter food needs of their cows. Each nursing cow eats about 27.5 pounds of food each day, and 3.5 pounds of that amount needs to be protein to promote the health of the calves. The Browns produce lots of hay on their farm, and the hay is about 5% protein. They plan to purchase some soybeans to mix with the hay to increase the amount of protein in their cows' diet. Soybeans consist of about 36% protein. About how many pounds of soybeans will they need to give each nursing cow each day?
  7. Marilyn said her family made two square pizzas at home, one 8" on a side and the other 12" on a side. She ate  $\frac{1}{4}$  of the small pizza and  $\frac{1}{6}$  of the larger pizza. So Marilyn says she ate  $\frac{5}{12}$  of the pizza.
    - a. How did Marilyn come up with this answer?
    - b. Make models that you could use to illustrate the situation.
    - c. Explain how you could use your models to find and explain the correct answer.
  8. A box contained 31 chocolates. The first day, Susan ate  $\frac{3}{4}$  of the number Jacqueline ate. The second day, Susan ate  $\frac{2}{3}$  of the number that Jacqueline ate that day, and the chocolates were all gone. How many chocolates did Susan eat?

## Level 4

1. **Cucumber Pomade.** Veal suet, 600 parts; lard, 1,000 parts; cucumber juice, 1,200 parts; tincture of tolu, 2 parts; rose water, 10 parts. To the liquefied suet and lard add the tolu tincture; when nearly cool, gradually incorporate the cucumber juice and rose water, previously mixed, stirring constantly. (A pomade is a greasy substance used to make hair look slick and shiny. Pomades were quite popular with men in the early 1900s. SOURCE: *Scientific American Cyclopedia of Formulas* published in 1913.)

Suppose that you make a batch of Cucumber Pomade using the 30 fluid ounces of veal suet that you have on hand. Assuming you have plenty of lard, cucumber juice, tincture of tolu, and rose water on hand, about how many fluid ounces of Cucumber Pomade can you make if you follow the directions above?

2. Which is bigger:  $\frac{10001}{10002}$  or  $\frac{100001}{100002}$ ? What more general truth does this example illustrate? Can you prove it?
3. Which is bigger:  $\frac{12345}{54321}$  or  $\frac{12346}{54322}$ ? What more general truth does this example illustrate? Can you prove it?
4. Fractions  $\frac{a}{b}$  and  $\frac{c}{d}$  are neighbor fractions if their difference  $\frac{ad-bc}{bd}$  has a numerator equal to  $\pm 1$ . Prove that
  - (a) in this case neither fraction can be simplified;
  - (b) if  $\frac{a}{b}$  and  $\frac{c}{d}$  are neighbor fractions, then  $\frac{a+c}{b+d}$  is between them and is a neighbor fraction for both  $\frac{a}{b}$  and  $\frac{c}{d}$ ; moreover,
  - (c) no fraction  $\frac{e}{f}$  with positive integer  $e$  and  $f$  such that  $f < b + d$  is between  $\frac{a}{b}$  and  $\frac{c}{d}$ .
5. A stick is divided by red marks into 7 equal segments and by green marks into 13 equal segments. Then it is cut into 20 equal pieces. Prove that any piece (except the two end pieces) contains exactly one mark (which may be red or green).
6. What is better, to get five percent of seven billion or seven percent of five billion?
7. How can you cut from a  $\frac{2}{3}$ -meter-long string a piece of length  $\frac{1}{2}$  meter, without having a meter stick?
8. Anton had a mug filled with milk and another mug with the same amount of tea. He took one teaspoonful of milk out of the first mug and stirred it into the second mug until it was thoroughly mixed. Next, he took one teaspoonful of the mixture in the second mug and stirred it into the first mug. Was there more milk in the tea or more tea in the milk?