SOUTHEAST OHIO MATH TEACHERS CIRCLE SUMMER IMMERSION 2016

SOLITAIRE MANCALA GAMES

June 6, 2016

1 HISTORICAL BACKGROUND

Mancala is a generic name for a family of $sowing$ games that are popular all over the world, particularly in Africa and parts of Asia. Archaeological evidence suggests that some games are at least 1300 years old. Description of many of the variations of Mancala have been recorded . A commercially available version that is often played in America is called Kalah. It was invented and patented by William Julius Champion Jr. in the $1950's$. Outside of the US, Oware (also known as Wari or Ayo) is perhaps the most widespread game in the Mancala family that is played on a 2×6 board. These games have also interested researchers in mathematics and machine learning. In this presentation, We will learn how to play simple solitaire mancala game called Tchoukaillon that facilitates the mathematical analysis of many of the other game variations. Tchoukaillon was introduced in 1977, and is derived from another Mancala variant called Tchuka Ruma that was first described in 1895.The

2 MATERIAL NEED TO PLAY TCHOUKAILLON

1. board consists of a sequence of bins that can contain stones. One of the end of the board are called Ruma which usually the size is bigger than the other bins.
2. Stones or Beads.

3 GOAL OF TCHOUKAILLON AND HOW TO PLAY

The object of Tchoukaillon is to move all of the stones originally on the board into the Ruma. During each turn, a player may pick up all of the stones in a selected bin and then sow them by depositing one stone in each succeeding bin towards the Ruma so that the last stone is deposited in the Ruma. Figure 3.1 illustrates a valid Tchoukaillon game on three bins, with the Ruma on the right side of the board.

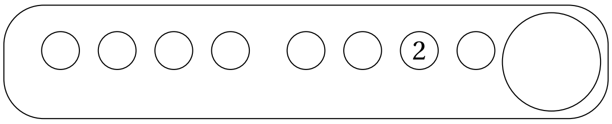

Figure 3.1: Not Solvable Board

A board is is not solvable if it is not possible to clear the board, for example the board in Figure 3.2 can not be solved.

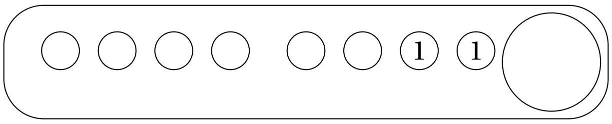

Figure 3.2: Not Solvable Board

4 LONGEVITY AND LENGHT OF TCHOUKAILLON BOARD

The number of steps needed to clear of a valid Tchoukaillon Board is called longevity, for simplicity we will write $\mathscr{L}$. Usually the longevity of the board may not be unique but in most of the case we will be only interested in the smallest one. The location of the foremost bin that contains beads are called length of the board, we just write L for it.

In Figure 4.1 we have a board in which $L = 1$ and after one step all beads (its one) end up in rumba, so $\mathscr{L} = 1$.

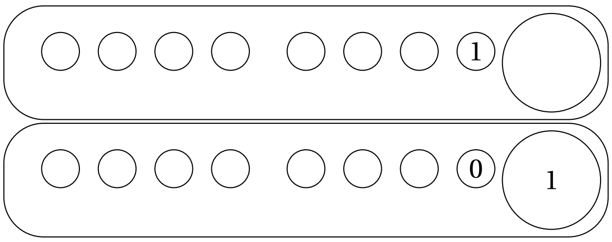

Figure 4.1: In this board $L = 1$ and $\mathscr{L} = 1$

In the Second example (figure 4.2) $L = 3$ and after 3 step the board will clear up of beads, hence $\mathscr{L} = 3$.

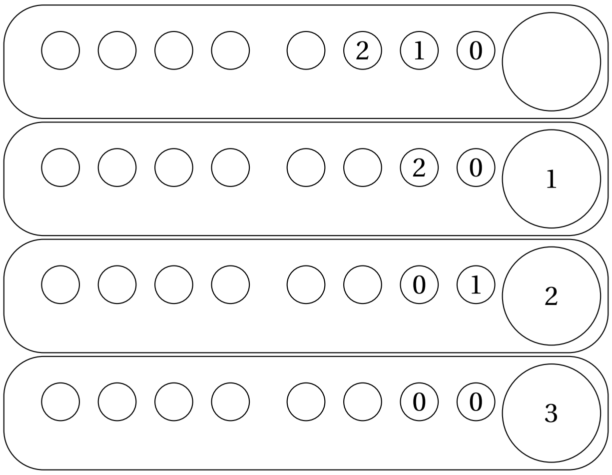

Figure 4.2: In this board $L = 3$ and $\mathscr{L} = 3$

Till this moment in the two previous example we have $L = \mathscr{L}$, the next example will show that is not true in general, we may have a case where $L < \mathscr{L}$

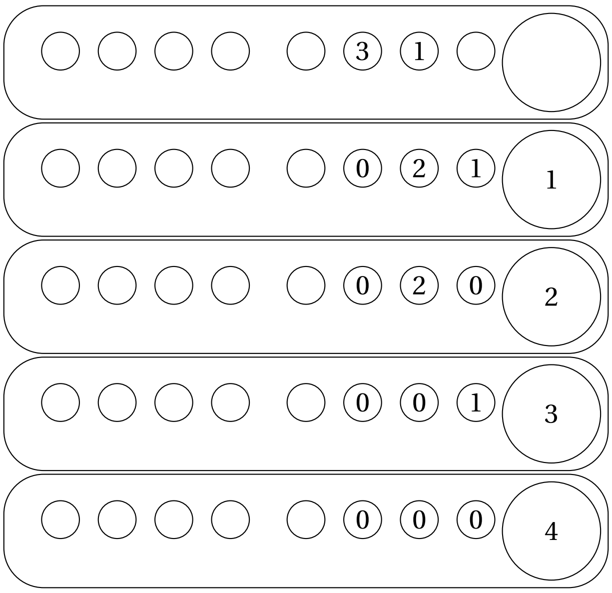

Figure 4.3: In this board $L = 3$ and $\mathscr{L} = 4$

1. Find $\mathscr{L}$ of each of the boards that given in the table below.

P6	P5	P4	P3	P2	P1	$\mathscr{L}$	L	n
0	0	0	0	0	0			0
0	0	0	0	0	1			1
0	0	0	0	2	0			2
0	0	0	0	2	1			3
0	0	0	3	1	0			4
0	0	0	3	1	1			5
0	0	4	2	0	0			6
0	0	4	2	0	1			7
0	0	4	2	2	0			8
0	0	4	2	2	1			9
0	5	3	1	1	0			10
0	5	3	1	1	1			11
6	4	2	0	0	0			12
6	4	2	0	0	1			13

2. Can you tell us what algorithm or strategiy you used for finding $\mathscr{L}$ in the last question?

3. If you have aborad that $L = n$ and $b_i = i$ that is $b_1 = 1, b_2 = 2, \cdots, b_n = n$ can you find $\mathscr{L}$? (is there is general formul that work for each n).

4. Can you find the unique winning board with n = 15 stones, without unplaying from the trivial board ((5,4,3,2,1) is trivial board)?

5. What is the smallest (greatest) possible number of stones for a Tchoukaillon board of length $\mathscr{L} = 6$?

Source: JONES, B; TAALMAN, L; TONGEN, A. Solitaire Mancala Games and the Chinese Remainder Theorem. American Mathematical Monthly. 120, 8, 706-724, Oct. 2013.