

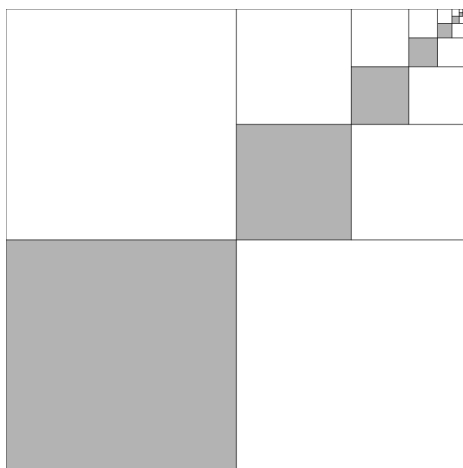
## Infinite Series Fraction Pictures

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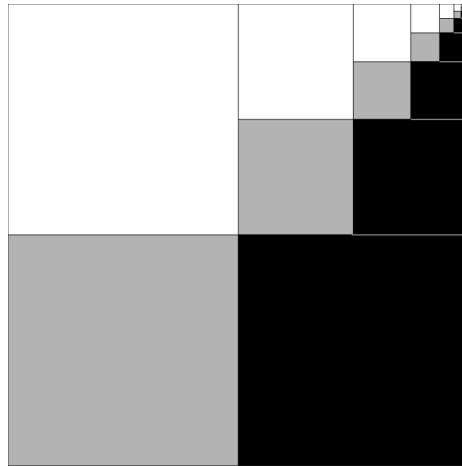
*Note: These notes were compiled by Bob Klein based on a session led by Amanda. Bob owns all of the mistakes and credits Amanda for all of the genius. UPDATE: Amanda says: "Zandra Vinegar should get the credit for the three opening examples (I took them from Nancy Blachman's recent compilation of excellent math problems with curricular connections)."*

Take a look at the picture below:



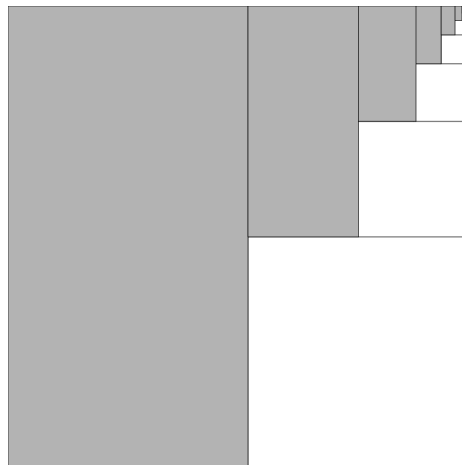
1. Without thinking, what would you guess is the area of all of the gray squares put together?
2. If the whole figure above has area 1, what is the area of the largest gray square? The next largest? The next?
3. If the smaller and smaller squares are grayed in as above, what is the total area grayed out in the figure above?
4. Can you write the sum (something + something + something +  $\dots$ ) for the total area?

Now look at the figure below:



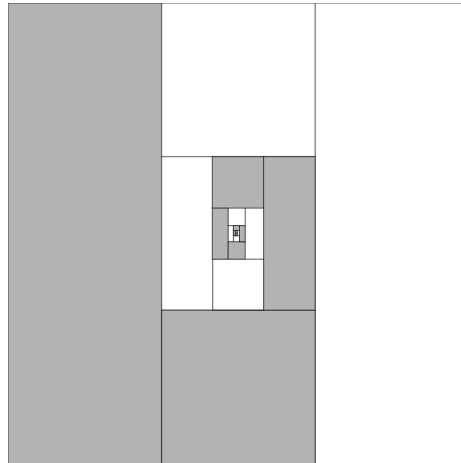
5. How does coloring the “bottom” squares black help to see the total area of just the gray squares? What is that area?
6. Is there another way to see that we get that sum without adding each gray area up? How many of each three squares of the same size is shaded?

Now consider a new shading of the unit square as shown below.

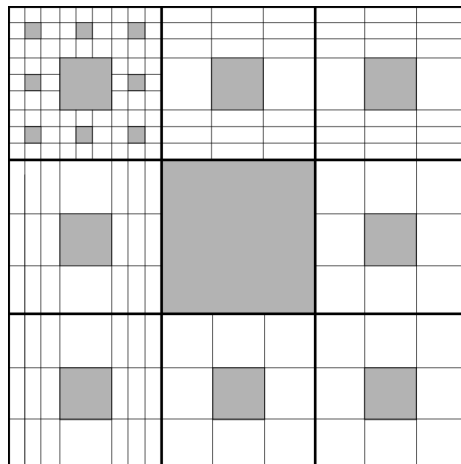


7. What is the total area of the shaded gray areas?
8. Can you write the sum (something + something + something +  $\dots$ ) for the total area?
9. How could you use the previous square (questions 1-4) to help answer this one?

10. What is the total area of the shaded gray area below?
11. Can you write the sum (something + something + something +  $\dots$ ) for the total area?



The square below is a little different.



The picture is incomplete. The pattern is to divide the big square into 9 congruent squares and to gray the center one. Then take each of the remaining white squares, divide it into 9 congruent squares and then gray the center squares for each of those. Repeat.

12. Without thinking, what would you “guess” the area is for the shaded square?
13. Can you write the sum (something + something + something +  $\dots$ ) for the total area?