# The Three Jugs Problem

You have an empty 3-gallon jug and an empty 5-gallon jug. You also have an 8-gallon jug that is full of water. You would like to give your friend exactly half of the water (4 gallons).

1. Can you think of a scheme that uses the three jugs to measure out exactly 4 gallons of water for your friend?

2. If you find a way do you think it's the way with the least number of steps? Or is there a shorter way?

3. Whether or not you find a way to measure 4 gallons of water, what amounts of water *can* you measure out? Keep track of the ones that work.

4. Consider the problem for different size jugs. For instance, suppose the empty jugs are 2-gallon and 6-gallon, and the full jug is 8 gallons? What amounts can you measure out in this scenario? What about if the jugs are 4, 6, and 10 gallons respectively? 5, 6, and 11?

5. Notice that in all of these versions of this problem the amount of water held by the two smaller jugs added together is the same as the amount of water held by the biggest jug. Try the problem for other size jugs for which this relationship also holds. Try to be systematic. See if you can find some patterns that help you predict which amounts can be measured with each combination of jugs. What are you noticing?

6. Are you figuring out any methods that help you measure out difficult amounts? It doesn't matter if these methods are the *fastest* way to measure out the amount. In fact you may have more luck finding one that always works if it's kind of long and repetitive.

7. In particular, have you found any maneuvers that can be repeated to measure out many different amounts?

8. Now suppose that like before you have an empty 3-gallon jug and an empty 5-gallon jug. But instead of having just one 8-gallon jug you have a whole garage full of full 8-gallon jugs. Can you measure out 4 gallons? Does it take the same number of steps as in the previous version of the problem? What about the other amounts you could measure in the first problem? Do these all work? Is your method similar? Describe how these two versions of the problem are similar.

9. Can you think of a maneuver that can be repeated that will eventually measure out all the amounts that can be measured in this scenario? Try the some of the other number combinations you worked on with the variation where you have a garage full of the largest size jug, all full of water. Does the variation change what amounts can be measured?
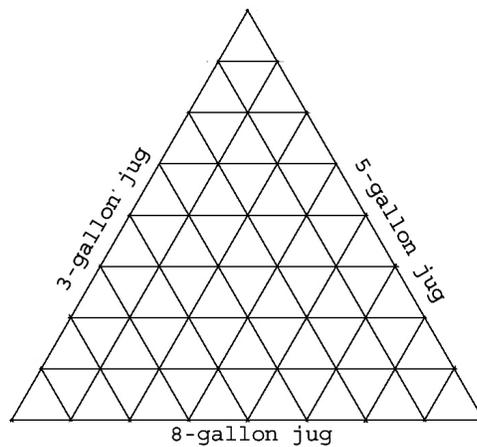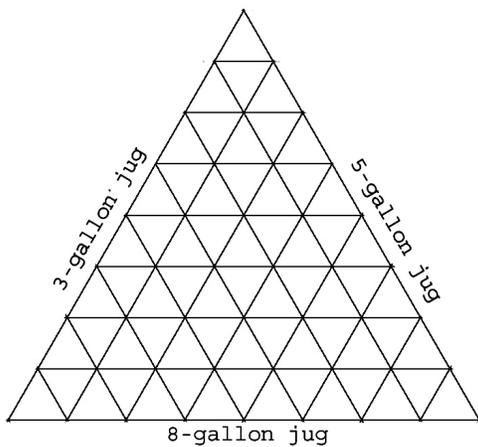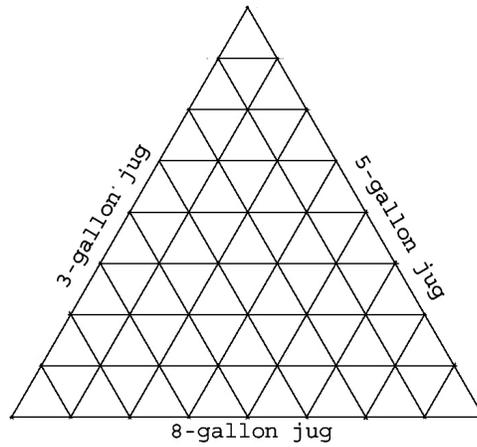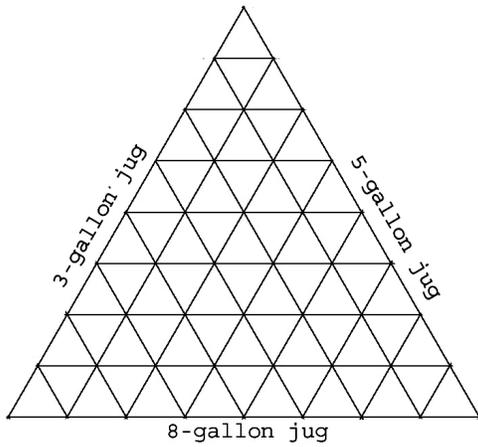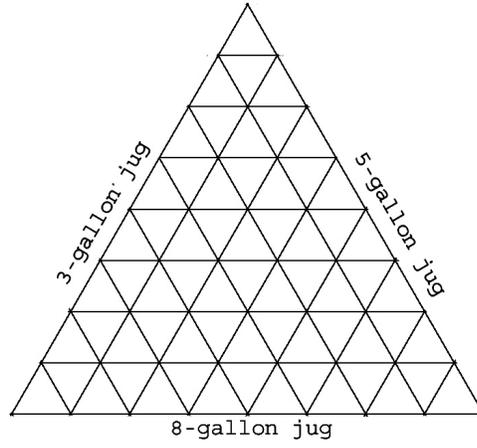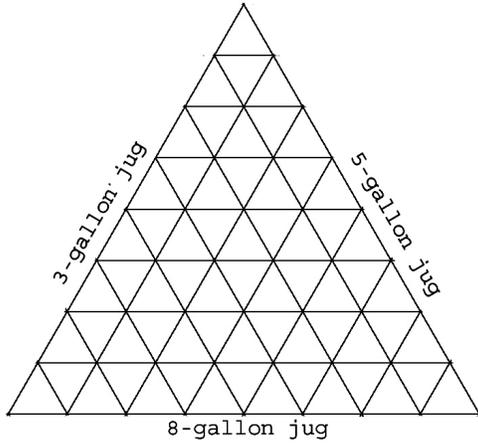
10. Now consider this picture:
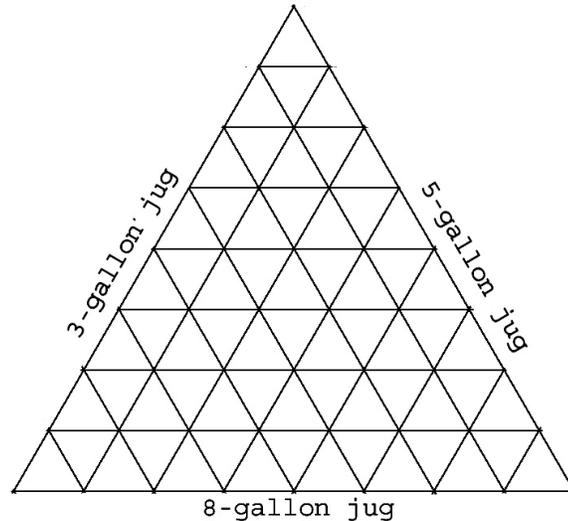
Suppose this triangle has a height of 8 units, so the height of each small triangle inside is 1 unit. Consider a point inside the triangle. Let its distance from the base of the triangle (if it were to fall right to the base) stand for the amount of water in the 8-gallon jug, the distance from the 3-gallon line be the amount of water in the 3-gallon jug, and the distance from the 5-gallon line stand for the amount of water in the 5-gallon jug.

11. What points in this triangle could represent a water distribution in the original problem? (either find some points that work, or, if you can, shade all the points that work).

12. Draw some of your solutions as connected points on this representation.

13. Can you use this kind of representation to explain how the problem works in general?

3-gallon jug   **5-gallon jug**   **8-gallon jug**

3-gallon jug   **5-gallon jug**   **8-gallon jug**

3-gallon jug   **5-gallon jug**   **8-gallon jug**

3-gallon jug   **5-gallon jug**   **8-gallon jug**

3-gallon jug   **5-gallon jug**   **8-gallon jug**

3-gallon jug   **5-gallon jug**   **8-gallon jug**