Polydrons

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Summer Workshop

Motivating Questions

- What is a polygon?
- What do you know about polygons?
- What is a regular polygon?
- What is a tessellation?
- What regular polygons will tessellate and WHY?

Polygon: a simple closed figure composed of straight line segments that lie in a plane.

The sum of the angles of a polygon with n sides will add to (n-2)180°.

To see this, divide each polygon into n-2 triangles by drawing all of the diagonals from one vertex.

Regular Polygon: a polygon in which all of the sides have the same length and all of the angles have the same degree measure.

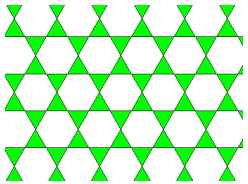
EACH angle of a regular polygon with n sides measures $(n-2)180°/n$.

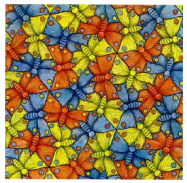
Angles in Regular Polygon

Number of Sides	Name	Sum of all angles	Each angle in regular polygon
3	Triangle	180°	60°
4	Quadrilateral	360°	90°
5	Pentagon	540°	108°
6	Hexagon	720°	120°
7	Heptagon	900°	128.5714…°
8	Octagon	1080°	135°
9	Nonagon	1260°	140°
10	Decagon	1440°	144°

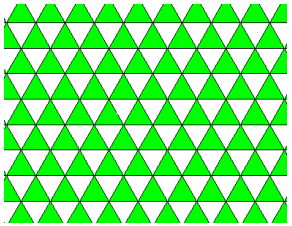
Tessellations

A tessellation is created when a shape is repeated over and over again covering a plane without any gaps or overlaps.





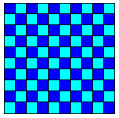
Regular tessellations use the same shape

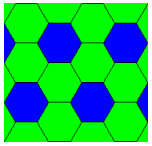


Which regular polygons will tessellate?

We know that at one point the sum of the angles needs to be 360°. We can easily determine which regular polygons have angle sizes that will accomplish this.

Figure	Size of angle	Number needed at one vertex
Triangle	60°	6
Square	90°	4
hexagon	120°	3





More Questions

- What is a platonic solid?
- How many platonic solids are there?
- Could there be more?
- How do you know?
- What is an Archimedean Solid?

Platonic Solid

- A three dimensional figure whose faces are congruent regular polygons and where the same number of faces meet at every vertex.
- If we look at one fixed vertex of the platonic solid there must be the same number of regular polygons meeting at that point. If there are only two regular polygons at one point they would collapse against each other and there would not be a three dimensional figure. Therefore, we must have at least 3 regular polygons at one point. Furthermore, the sum of the angles at that point must be less than 360°.

Using Triangles – each angle 60°

- 3 triangles at one point – Tetrahedron
- 4 triangles at one point – Octahedron
- 5 triangles at one point – Icosahedron
- 6 triangles at one point – NOT POSSIBLE!

Using Squares – each angle 90°

- 3 squares at one point – Hexahedron
- 4 squares at one point – NOT POSSIBLE!

Using Pentagons – each angle 108°

- 3 pentagons at one point – Dodecahedron
- 4 pentagons at one point – NOT POSSIBLE!

Using Hexagons – each angle 120°

- 3 hexagons at one point – NOT POSSIBLE!
- In fact, no regular polygon with more than five sides can be used to make a platonic solid. As the number of sides of a regular polygon increases so does the size of the angle. Since 3 x 120° = 360° we can see that a platonic solid will not be formed.

Solid	#polygons at one vertex-M	#vertices in one polygon-N	Total# Faces-F	Total Vertices V	Total Edges E
Tetrahedron	3	3	4	4	6
Hexahedron	3	4	6	8	12
Octahedron	4	3	8	6	12
Dodecahedron	3	5	12	20	30
Icosahedron	5	3	20	12	30

$E=NF/2$ $V=NF/M$ $F+V=E+2$ $F=4M/2M-MN+2N$

Conclusion?

There are only five Platonic Solids!

BUT.....

What is an Archimedean Solid???