Problems for South East Ohio Math Teachers' Circle
Compiled by Judith Covington

1. There were 60 students in the gym. Three-fourths of them were seniors. After a group of seniors left to attend a meeting, only two-fifths of the remaining students were seniors. How many students left to attend the meeting?

2. Two algebra classes took the same exam. The mean grade for the first class was 88. The second class had a mean grade of 79. The mean grade of the two classes combined was 84. If there were 25 students in the first class, how many were in the second class?

3. Find all numbers less than 1000 that yield a remainder of 4 when divided by 5, a remainder of 7 when divided by 9, and a remainder of 9 when divided by 11. How do you know you have found all of them?

4. A ditto number is a six-digit number formed by repeating a three-digit number. These are some examples of ditto numbers: 368368; 107107; 499499; etc. True or False: All ditto numbers are multiples of 13. (Be prepared to convince us that your answer is correct.)

5. At various times the boss gives the secretary a letter to type, putting it on top of the secretary's in-box. The secretary types the top letter. If there are five letters, and the boss delivers them in the order 1 2 3 4 5, which of the following could not be the order in which the secretary types them?
   a. 1 2 3 4 5 b. 2 4 3 5 1 c. 3 2 4 1 5 d. 4 5 2 3 1 e. 5 4 3 2 1

6. A contractor knows that one of his bricklayers would take 9 hours to build a chimney, and his other bricklayer would take 10 hours to do the job. He also knows that when they work together they talk a lot, and their combined output is decreased by 10 bricks per hour. But since he is in a hurry to have the chimney built, he assigns both men to the job. They complete the job in 5 hours. How many bricks are in the chimney? ("Algebra" is not needed.)

7. How many zeroes will appear at the end of the value of 45! (45x44x43 … x4x3x2x1) ?

8. Little Joey has nickels, dimes, and quarters in his piggy bank. One day he poured them all out on the dining room table and amused himself counting and arranging the coins. First he put them into seven equal numbered piles, with one pile containing only quarters and all of the quarters. Next, he arranges the coins in groups of five with exactly two dimes in each group. Finally, he counted the nickels, and there were 64. What is the total value of all Joey's coins?

9. Martha is the chief hippopotamus caretaker at the Wild Animal Park in San Diego, California. She has just arrived at the cargo dock in the downtown harbor to pick up four members of the endangered species hippopotamus mathematicus recently rescued from African poachers. To complete the paperwork she needs to weigh them, but the only scale big enough to weigh a hippo is the truck scale that starts at 300 kilograms – more than any of the hippos weigh! Martha is puzzled for a few minutes, then gets the idea of weighing them in pairs. She thinks that if she gets the weight of every possible pair, she can figure out later the weights of the individual hippos. She measures the weights pair by pair, getting 312, 356, 378, 444 and 466 kilograms. As she tried to weigh the heaviest pair of hippos, the scale breaks.

   a) What was the weight of the last pair of hippos that broke the scale?
   b) What are the weights of all the individual hippos? Are you sure?

10. Take a three digit number with distinct digits. Now, reverse the digits and take the positive difference between your two numbers. Now, take your answer, reverse the digits. Add the two numbers. What answer do you get? Will you always get the result? Explain how you know.

11. There were thousands of people in a medical research program. They were assigned to two equal-numbered groups. The ratio of men to women in Group I was 5 to 3; and in Group II, that ratio was 2 to 3. What percent of all the people in the study were women?

12. Three boxes are each filled with billiard balls. One contains only even numbered balls, one contains only odd numbered balls, and the third box contains a mixture of odd and even balls. All three boxes are mislabeled. You partly open the top flap of one box, so that you can see the number on only one ball. With this information, can the three boxes be correctly labeled as ODD, EVEN, and MIXED?

13. A giant watermelon weighed 50 pounds and was 99 percent water. After sitting in the hot sun, some of the water evaporated so that the watermelon was only 98 percent water. What is the new weight of the watermelon?

14. Ninety nine girls and one boy are in a mathematics class. How many girls must leave the room so that the percentage of girls in the room becomes 96 percent?

15. Two janitors at Central Middle School entertained themselves with the 250 lockers along the main corridor the evening after the last day of school. The first janitor opened every locker. The next janitor closed every second locker. The third time, the janitor changed the state (open or closed) of every third locker. This process continued, with the state of every nth locker being changed during the nth pass, until the janitors has passed down the corridor 250 times. In the end, which lockers were open?

16. A cottage heated by wood was shared for the night by three people, Larry, Mo, and Curly. Larry brought 5 logs for the fire, Mo bought 3 logs for the fire and Curly did not bring any logs. Instead, Curly brought $8 to pay for his share. All the logs were burned that night, so how should Larry and Mo split up the $8 so that each person contributed his share?

17. A certain number has exactly 12 factors. If two of these factors are 27 and 7, name two possible values of the number. Are there other possibilities?

18. Jeff had fewer than 1000 blocks. When he made five equal rows, he had one left over, with four equal rows he had one left over, and with nine equal rows, he had none left over. How many blocks might Jeff have? (Give all possible answers)

19. Heidi keeps track of her scores when she plays her favorite board game. The mean of her six best scores was 243 until today, when she replaced her sixth-best score with 237. If the new mean of her six best scores is 246, by how many points has her sixth-best score increased?

20. The average value of all the pennies, nickels, dimes, and quarters in Sam's wallet is 20 cents. If he had one more quarter, the average would be 21 cents. How many dimes does he have?

21. Jill can mow her mother's yard in 90 minutes, while it takes her brother only 60 minutes. How many minutes would it take them to mow the yard if they worked together using two mowers? (Can you solve without algebra?)

22. A sequence of nine numbers is formed according to this rule: Each number, after the first two, is the sum of the two numbers immediately preceding it. The first number in the sequence is 7, and the middle number is 2. What is the last number?

23. I have some change (pennies, nickels, and dimes) in my pocket. Forty-two of the coins are dimes, 25% of them are nickels, and five-ninths of them are pennies. What is the total value of my change?

24. If Pat is 25% taller than Lee, then Lee is what percent shorter than Pat?

25. Explain how to list the following numbers in ascending order: $3^{14}, 4^{9}, 8^{7}, 16^{5}$ without using a calculator.

26. Keith has two glasses each containing some orange juice mixed with water. The ratio of orange juice to water in the first glass is 1:1. The ratio of orange juice to water in the second glass is 1:3. The second glass is three times as large as the first glass. If the two glasses are mixed together, what will be the ratio of orange juice to water in the resulting mixture?

27. Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

28. Jay is taking a test that has 30 questions. He earns 9 points for every correct answer and loses 5 points for every wrong answer. If Jay answered every question and his score was 18, how many questions did he answer correctly on the test?

29. There are 80 M&M's in a bag, and 4/5 of them are yellow. After Suzy ate some of the yellow ones, only ¾ of the remaining candies were yellow. How many M&M's did Suzy eat?