Mad Veterinarian

1. A mad veterinarian has invented an animal transmogrifying machine. If you put in two cats or two dogs, then one dog comes out of the machine. If you put in one cat and one dog, then one cat comes out. The veterinarian’s goal is to end up with only one cat and no other animals. For example, you might start with three cats and a dog. What happens in this game? (Can the veterinarian win?) What if the veterinarian starts with a different collection of animals? (Generalize the solution as much as possible.) Can you connect this to something basic from mathematics?

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2. The veterinarian’s old machine breaks. Now the veterinarian has dogs, cats, and mice. The new transmogrifying machine can take any two different animals and then out comes the third animal. Can the veterinarian achieve his goal of ending up with just one cat if he starts with three cats and one dog? What about other starting situations, like four of each animal for example? What if he can reverse the rule when he wishes, putting in one animal and having one of each of the other two animals come out?

3. One hundred dwarves are caught by a witch. They are told that in a minute they will be positioned in a row, each dwarf facing the ones in front of him but not able to see the one behind him. They will each get a hat, which will be either red or blue. (No dwarf will see his own hat, and each dwarf will only see the hats in front of him.) Starting with the dwarf in the back (who can see all the hats in front of him) and proceeding in order, each dwarf will guess the color of his own hat. If he is right, he gets released immediately; otherwise he becomes the witch’s servant for life.

The dwarves have one minute to think of a strategy before the fun begins. Once they are all placed and have their hats on, they are not allowed to say anything but “red” or “blue” and strictly in the order from the back to the front. How many dwarves can be liberated and how?

Of course, the dwarves are not allowed to send any signals. The only means of conveying information is to make a guess that everybody can hear.

4. Given some set of numbers (say, 2, 3, 5, 7) written on the board, erase any two of them (say, x and y) and write their sum, x+y. What happens in the long run?

5. What if, in the previous problem, x+y is replaced by xy? What if it is replaced by xy+x+y?

6. On a planet far away, the only inhabitants are chameleons. They come in three colors: green, yellow, and red. Further, on this planet there is a law which governs how chameleons can change their color. Whenever two chameleons of different colors meet, they both change to the third color. Given the initial number of chameleons of each color (say, 4 green, 5 yellow, and 5 red), is it possible to have all chameleons change to the same color? Try some other intital numbers; what can you conclude?